Numericke metode resavanja jednacina kretanja, u Ojlerovskoj formulaciji u 35 slika

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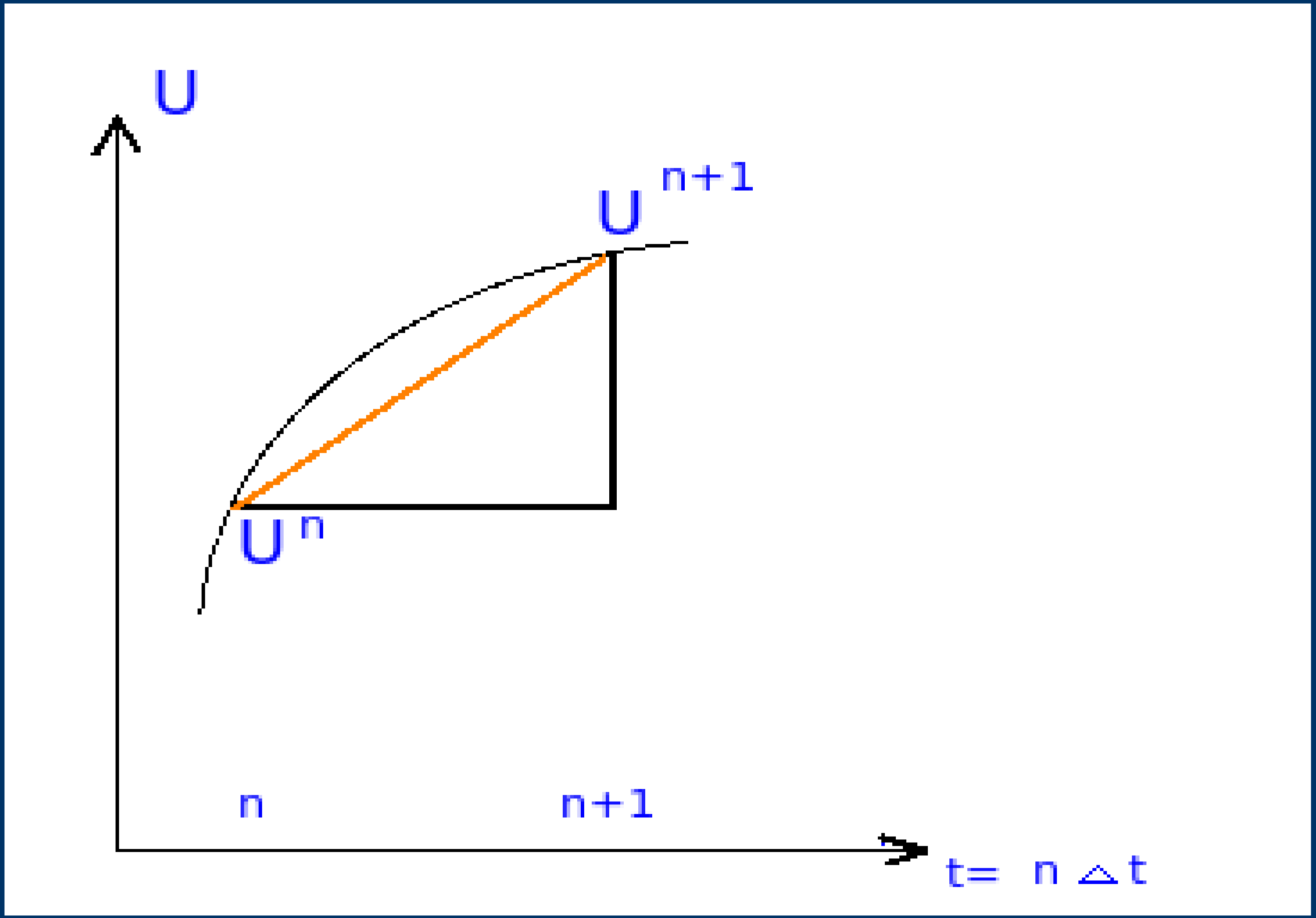
Numericko diferenciranje u vremenu u [17] slika

- Vreme postaje diskretna promenljiva
-
- Integraljenje u vremenu postaje “koracanje” u vremenu.
-
- Uz tacnost druga i jednako vazna karakteristika je efikasnost :
- sa sto *manje* koraka zavrsiti integraciju, tzv
  -
    - *racunska efikasnost*

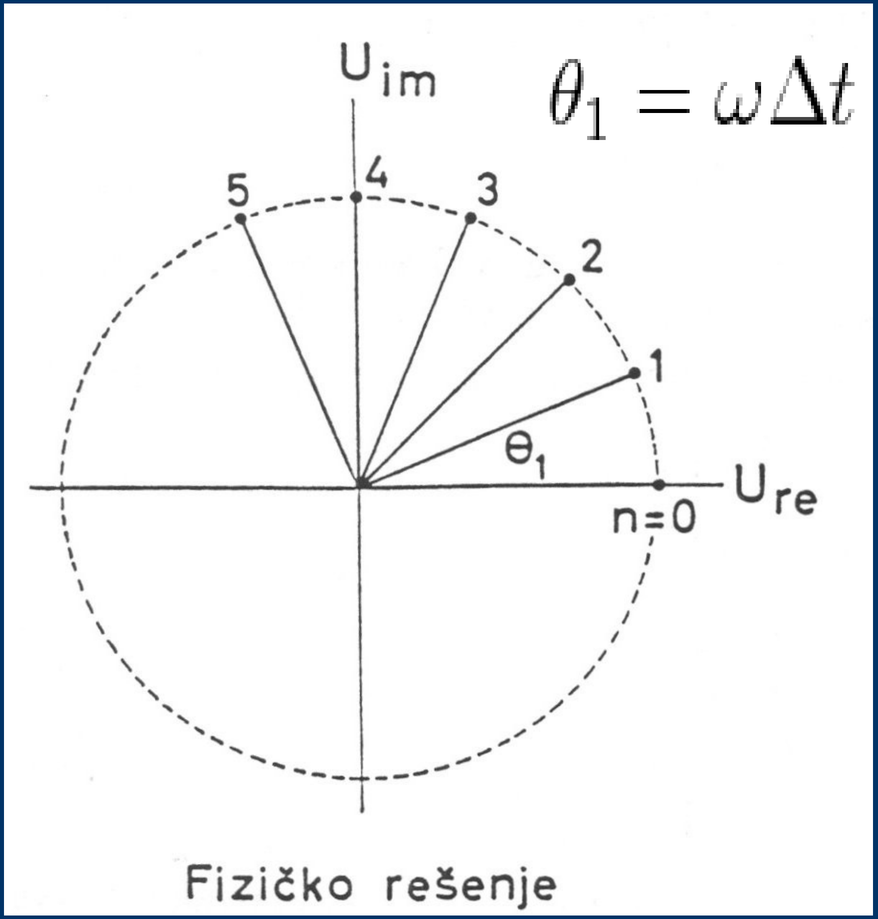
Uz aproksimiranje diferenciranja u vremenu



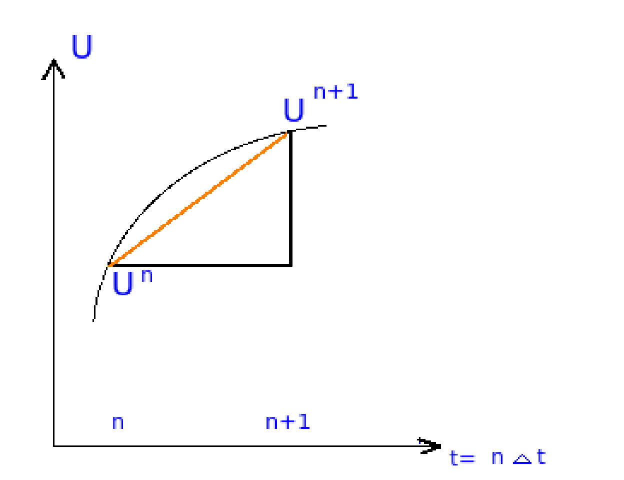
Diferencijalna jednacina oscilacija, njeno analiticko resenje i graficki prikaz

$$\frac{dU}{dt} = \mathbf{i}\omega U$$

$$U(t) = U(0)e^{\mathbf{i}\omega t}$$



Neiterativne sheme



-
- Eksplicitne
-

$$\frac{U^{n+1}-U^n}{\Delta t} = \mathrm{i}\omega U^n \qquad U^{n+1} = U^n + \mathrm{i}\omega t U^n = U^n + \mathrm{i}p U^n$$

-
- Implicitne

$$\frac{U^{n+1}-U^n}{\Delta t} = \mathrm{i}\omega U^{n+1} \qquad U^{n+1} = U^n + \mathrm{i}\omega t U^n = U^n + \mathrm{i}p U^{n+1}$$

$$\frac{U^{n+1}-U^n}{\Delta t} = \mathrm{i}\omega \left(\frac{1}{2}U^{n+1} + \frac{1}{2}U^n\right)$$

Von-nojman pristup analizi stabilnosti vremeskih shema

- Uslov
- $\|U^{n+1}\| < M$
- uz
- $\lambda = 1 + \delta$
- postaje

$$\delta \sim \Delta t$$

- Definicija faktora pojacanja

$$\lambda = \frac{U^{n+1}}{U^n}$$

Faktori pojacanja za neiterativne sheme sa dva nivoa

- Ojler
-
- $\lambda_{Ojler} = 1 + \mathbf{i}p$
-
- Unatrag
-

$$\lambda_{Unutrag} = \frac{1-\mathbf{i}p}{1+p^2}$$

-
- Trapezoidna shema
-
- $$\lambda_{Trapez} = \frac{1-\frac{1}{4}p^2+\mathbf{i}p}{1+\frac{1}{4}p^4}$$

Problem sa implicitnimshemama

- Analiticka forma j. za plitku vodu
-
- $$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} \quad ; \quad \frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$
-
-
- Jedna moguca implicitna aproksimacija

$$\frac{u^{n+1}-u^{n}}{\Delta t} = \left[-g\frac{\partial h}{\partial x}\right]^{n+1} \quad ; \quad \frac{h^{n+1}-h^{n}}{\Delta t} = \left[H\frac{\partial u}{\partial x}\right]^{n+1}$$

Iterativne sheme

$$\frac{U^* - U^n}{\Delta t} = \mathbf{i}\omega U^n$$

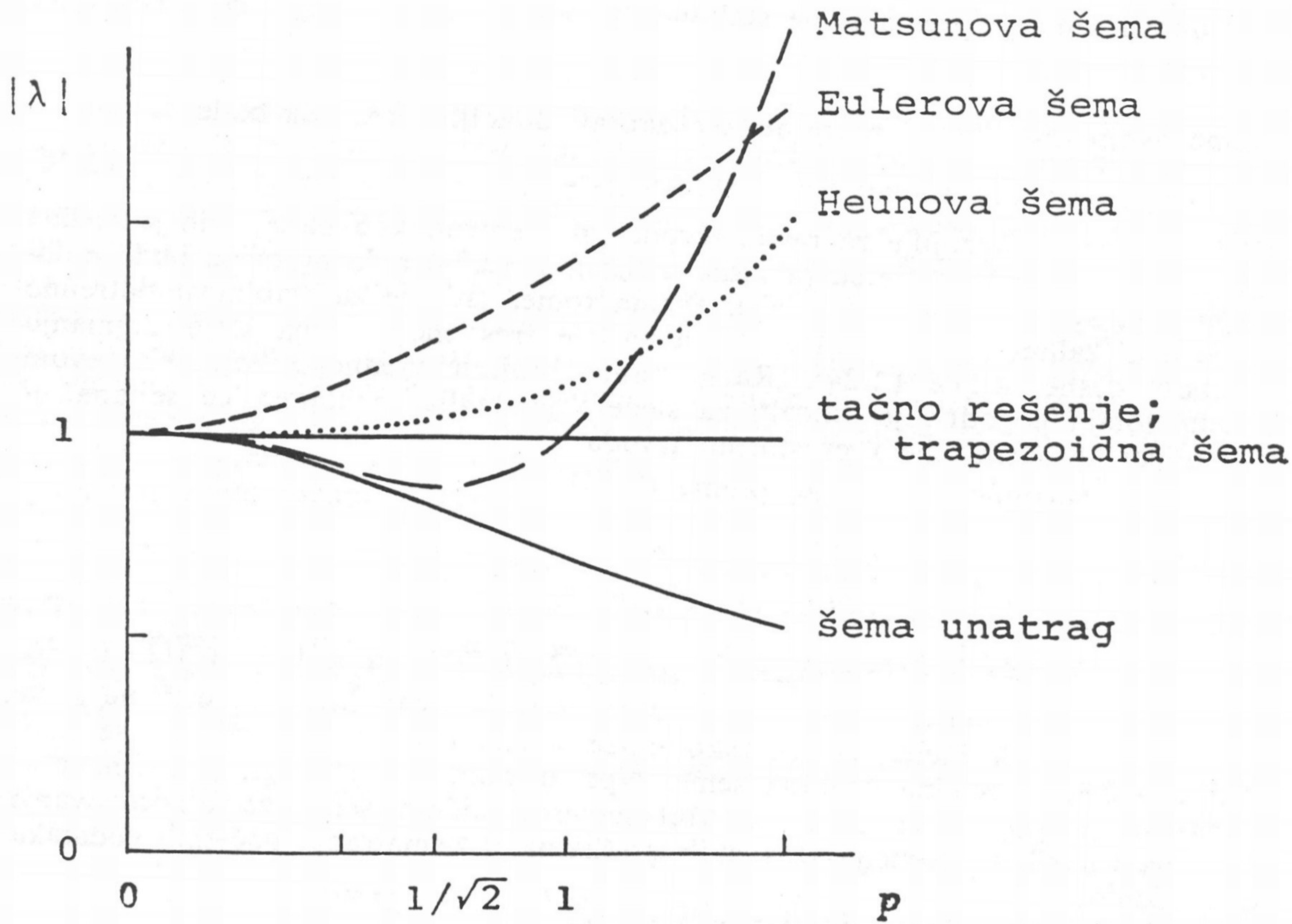
$$\frac{U^* - U^n}{\Delta t} = \mathbf{i}\omega U^n$$

$$\frac{U^{n+1} - U^n}{\Delta t} = \mathbf{i}\omega U^*$$

$$\frac{U^{n+1} - U^n}{\Delta t} = \mathbf{i}\omega \left(\frac{1}{2}U^* + \frac{1}{2}U^n\right)$$

$$\frac{u^* - u^n}{\Delta t} = \left[-g\frac{\partial h}{\partial x}\right]^n \quad ; \quad \frac{h^* - h^n}{\Delta t} = \left[H\frac{\partial u}{\partial x}\right]^n$$

$$\frac{u^{n+1} - u^n}{\Delta t} = \left[-g\frac{\partial h}{\partial x}\right]^* \quad ; \quad \frac{h^{n+1} - h^n}{\Delta t} = \left[H\frac{\partial u}{\partial x}\right]^*$$



Sl. 2.1. Faktor povećavanja $|\lambda|$ u zavisnosti od $p \equiv \omega \Delta t$ za pet razmatranih šema sa dva nivoa.

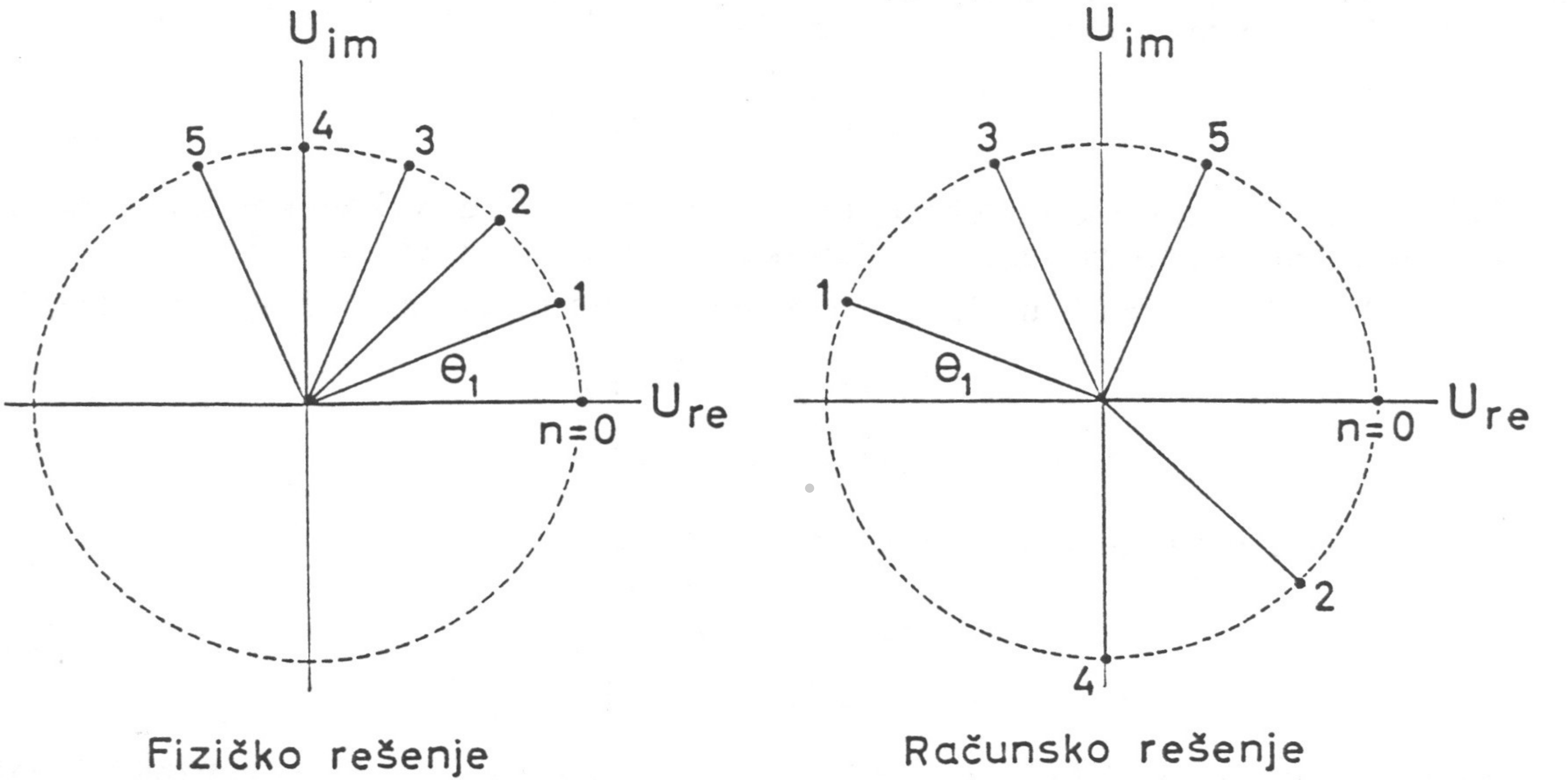
Preskocna shema

$$\frac{U^{n+1} - U^{n-1}}{\Delta t} = \mathbf{i}\omega \cdot U^n$$

•

$$\lambda_{1,2}^{Pres} = \pm\sqrt{1-p^2} + 2\mathbf{i}p$$

Fizicko (desno) i numericko (levo) resenje kod primene preskocne seme



Sl. 2.3. Položaj fizičkog i računskog rešenja u kompleksnoj ravni, kod preskočne šeme, za slučaj kada je $\theta_1 = \pi/8$ i kada rešenja u početnom momentu imaju samo realni deo, za razne vrednosti *n*.

Adams-Bashforth-ova shema

$$\frac{U^{n+1}-U^n}{\Delta t} = \mathbf{i}\omega \cdot \left(\frac{3}{2}U^n - \frac{1}{2}U^{n-1}\right)$$

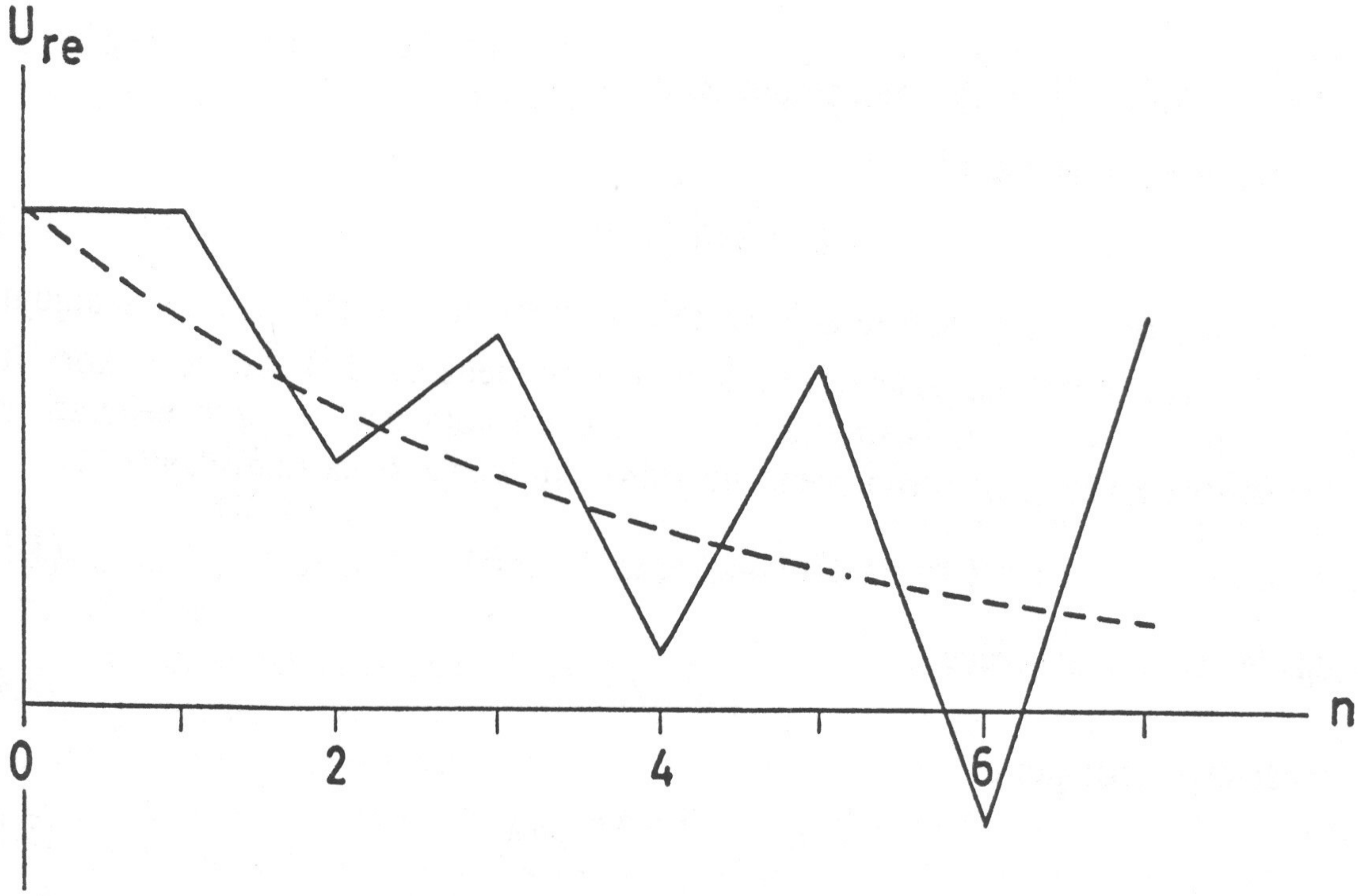
$$\lambda_1^{AB} = \qquad \lambda_2^{AB} =$$

Jednacina treanja

- Jednacina
- $$\frac{\partial U}{\partial t} = -\kappa U$$
-
-
- analiticko resenje
- $$U(t) = U(0)e^{-\kappa t}$$
-

Faktor pojacanja kod jednacine trenja

$$\lambda < 1$$



Sl. 3.1. Primer za nestabilnost preskočne šeme kod primene na jednačinu trenja.

Prigusene oscilacije

- Jednacina prigusenih oscilacije
-
- $$\frac{dU}{dt} = \mathbf{i}\omega U - \kappa U$$
-
-
- Analiticko resenje

$$U(t) = U(0)e^{-\kappa t} \cdot e^{\mathbf{i}\omega t}$$

Numericko resavanje jednacine prigusenih oscilacija

- Koristicemo metod “rasceplivanja”, tj. posebno tretirati “osc.” clan a posebno clan prigusenja

$$\frac{U^p - U^{n-1}}{2\Delta t_1} = \mathbf{i}\omega \cdot U^n$$

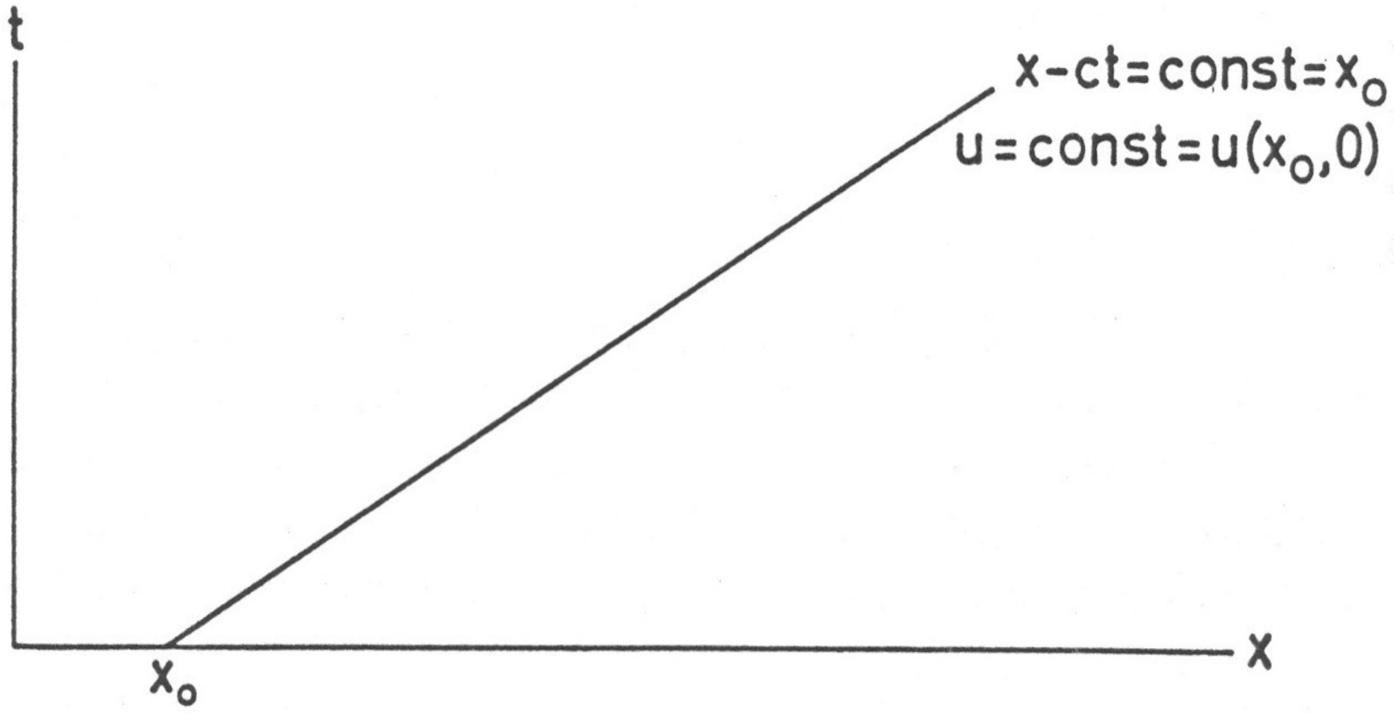
$$\frac{U^{n+1} - U^p}{\Delta t_2} = -\kappa \cdot U^n$$

Numericko diferenciranje u prostoru

Karakteristike anal. Resenja ADV jed.

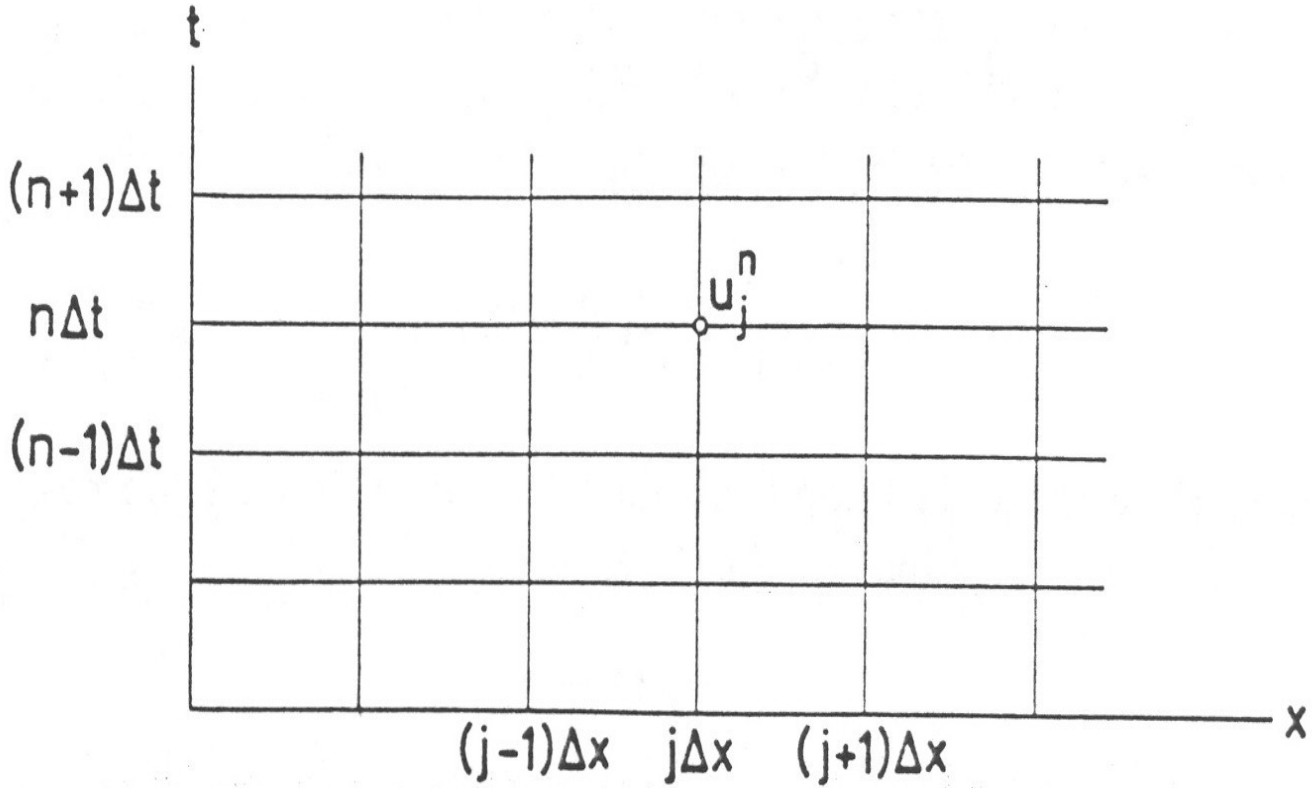
$$\frac{\partial f}{\partial t} + c \cdot \frac{\partial f}{\partial x} = 0$$

$$f(x,t) = F(x - ct)$$

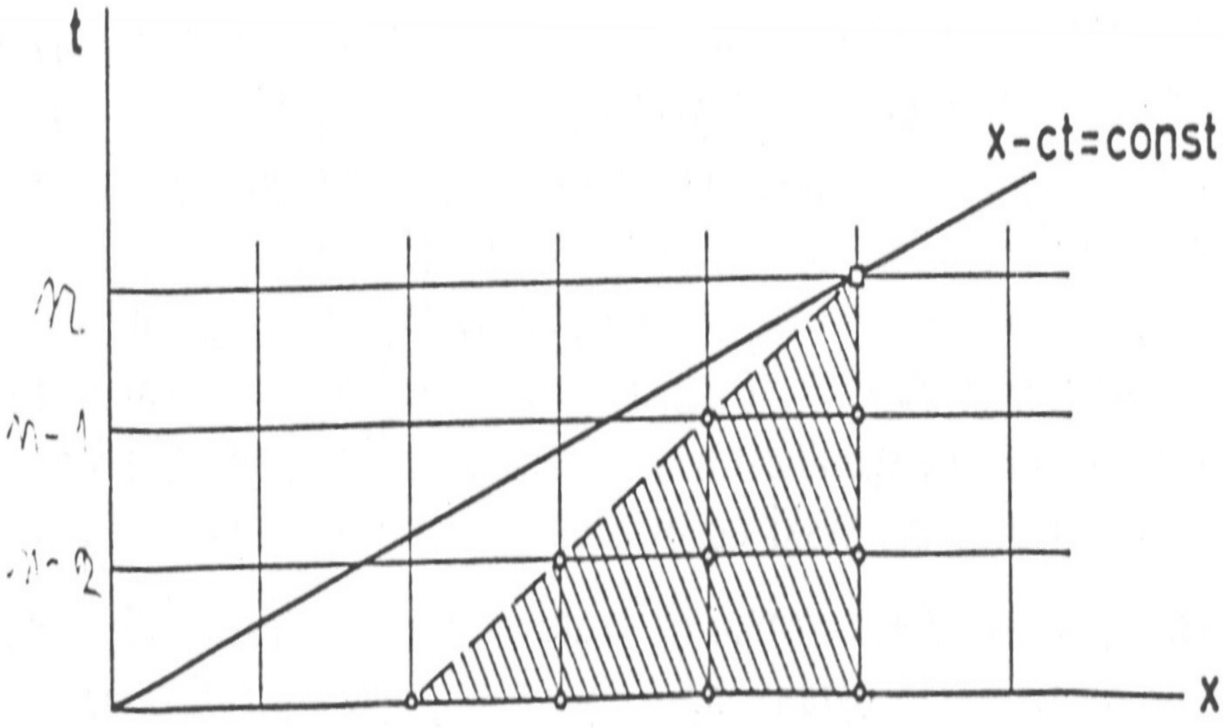


Sl. 3.1. Jedna od karakteristika linearne advektivne jednačine (3.1).

Diskretizacija u vremenu i prostoru



Sl. 3.2. Mreža za približno rešavanje jednačine (3.1).



Sl. 4.1. Moguć položaj karakteristike tačne jednačine i oblasti zavisnosti numeričkog rešenja advektivne jednačine.